Translations And Homogenous Coordinates

CS 476 Ursinus College

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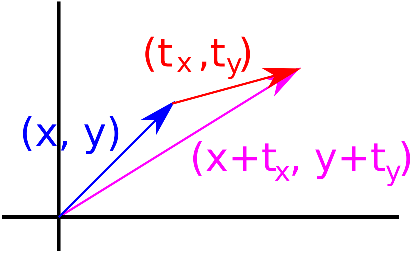
Translation Matrix

$$f((x, y)) = (x + a, y + b)$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$



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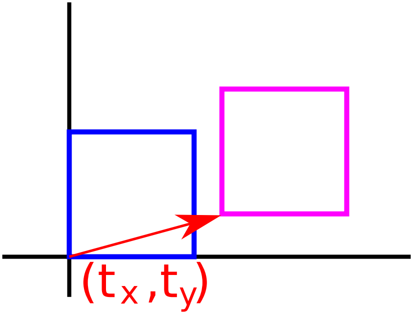

Translation Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

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for any a, b, c, d. Translation is a *nonlinear* operation

Translation Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

But this is impossible!!

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for any a, b, c, d. Translation is a *nonlinear* operation

Homogenous Coordinates

Pure translation with homogenous coordinates

$$\begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix} \\ 1 \end{bmatrix}$$

Homogenous Coordinates

General 2D transformation + translation

$$\begin{bmatrix} a & b & T_x \\ c & d & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix} \\ 1 \end{bmatrix}$$

We have some extra baggage, but we have more freedom now